Condensation of Planckian Modes and the Inflaton

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Abstract

To confront the transplanckian problem encountered in the backward extrapolation of the cosmological expansion of the momenta of the modes of quantum field theory, it is proposed that there is a reservoir, depository of transplanckian degrees of freedom. These are solicited by the cisplanckian modes so as to keep their density fixed and the total energy density of vacuum at a minimum. The mechanism is due to mode - reservoir interaction, whereupon virtual quantum processes give rise to an effective mode-mode attraction. A BCS condensate results. It has a massless and massy collective excitation, the latter identified with the inflaton. For an effective non dimensional mode-reservoir coupling constant, $g \simeq 0.3$, the order of magnitude of its mass is what is required to account for cosmological fluctuations i.e. $\mathcal{O}(10^{-6}–10^{-5})m_{Planck}$.

The establishment of the inflaton field scenario as the mechanism of inflation must be counted as one of the outstanding successes of physics in recent years [1, 2]. Whereas the classical inflation was originally introduced [3, 4] to implement the slow roll mechanism of inflation thereby eliminating the flaws of the earlier inflationary proposals [5] which were introduced to provide for a causal big bang and to explain flatness, it is now clear that the inflaton must be regarded as a quantum field in order to give a correct quantitative account of cosmological fluctuations [1, 2].

Yet, this quantum field seems to play no rôle in physics other than to implement inflation. Its mass, μ, is estimated to be $(10^{-6}$–$10^{-5})m_{Pl}$ [1] (m_{Pl}=Planck mass), so if it does play any rôle in particle physics it will have to be in the context of one of the conjectured hypotheses current in the field e.g. grand unification or supersymmetry.

In this essay, we shall exhibit an alternative possibility wherein the inflaton emerges as a collective excitation of quantum gravity. Its mass differs from the natural scale, m_{Pl}, by a sensitive reduction factor so that it is possible to have $\mu << m_{Pl}$ without undue tuning of parameters.

It is commonly admitted that quantum field theory (QFT), expressed in terms of modes in interaction, becomes inoperative for $|\vec{k}| > \mathcal{O}(m_{Pl})$; $k =$ momentum of the mode. For beyond the planckian scale, gravitation dominates. In this we adhere to a non covariant Hamiltonian formalism wherein $\vec{k}$ is the eigenvalue of the translation operator on a standard space-like homogeneous surface of our evolving universe i.e. our frame is our universe (We hope in the future to develop a covariant formulation).

The difficulty is that $|\vec{k}|$ scales like a^{-1} (a = scale factor). Naïvely, this would imply a reduction of the cut-off proportional to a^{-1}, say $\mathcal{O}(10^{-50})$ from the beginning of inflation to the present. A planckian cut-off today would place us inadmissably into the transplanckian region in the past.

Furthermore, one believes that microscopic physics should be but slightly influenced by the macroscopic expansion, there being a factor of 10^{40} between the present Hubble radius and that of the proton. The physics of matter is not conformally invariant. Therefore QFT in the past should not differ sensibly from QFT in the present. Nor should the cut-off.

To accomplish this, we propose that the degrees of freedom of fields fall into two classes, the cisplanckian modes of QFT and the infinite remainder comprising a transplanckian reservoir. We may think of two fluids. They are in interaction in such manner that the QFT modes are solicited from the reservoir during the expansion so as to keep their density (i.e. the cut-off) fixed. In this way the total energy density of vacuum is maintained at its minimum, as befits the vacuum state.

The concept of a transplanckian reservoir may seem unconventional, but it is in fact familiar, for example in string theory. Here the string degrees of freedom in the massless sector, cut off at m_{Pl}, comprise the stuff of usual QFT, and the infinite remainder are in the transplanckian modes, presumed to take up black hole configurations [6], perhaps modeling Wheeler's foam.

In this essay, however, we do not adopt a specific model of the reservoir, but treat it as a phenomenological black box. This is sufficient to deliver an inflaton and a formula for its mass, essentially in terms of one dimensionless parameter, (in addition to its natural scale, m_{Pl}) in attendance of a quantitative realization of a precise model.

The very existence of a stable vacuum points to the existence of a transplanckian reservoir. Indeed, since the zero point energy density of QFT in the presence of a cut-off scales like a^{-4}, the fluid of modes will exert a "zero point pressure" resulting in its infinite dilution in the absence of a compensating "internal pressure". Therefore there must be an attractive interaction between modes and the reservoir. (This consideration will ultimately lead to some understanding of the cosmological constant, Λ, but that is not the content of this paper).

The mode-reservoir interaction gives rise to mode-mode attraction through the intermediary of the reservoir in virtual processes (analog of phonon exchange among superconducting electrons). The consequent Cooper pair formation leads to a BCS condensate [7]. Its massy, collective excitation [8] is identified with the inflaton.

We now draw up a simple mathematical expression of these ideas. We focus on mode-reservoir interactions since it is these that lead to mode-mode attraction, whence a condensate and the inflaton. Usual QFT, which does not allow for variation of mode number, is not available for this task.

The interactions are taken to be of two sorts :

$$\rho_{-\vec{q}}\ a^{\dagger}_{\vec{k}+\vec{q}}\ a^{\dagger}_{-\vec{k}} + h.c. \tag{1a}$$

$$\rho_{-\vec{q}}\ a^{\dagger}_{\vec{k}+\vec{q}}\ a_{\vec{k}} + h.c. \tag{1b}$$

$\rho_{\vec{q}}$ is the q^{th} Fourier transform of the reservoir density. Operators $a_{\vec{k}}$ ($a^{\dagger}_{\vec{k}}$) annihilate (create) modes $\vec{k}$. They have the properties $a^2_{\vec{k}} = 0$; $n^2_{\vec{k}} = n_{\vec{k}}$ (where $n_{\vec{k}} = a^{\dagger}_{\vec{k}}\ a_{\vec{k}}$) whence its eigenvalues are 0, 1. This is required because a mode either is or is not. There is no need in what follows to specify commutation rules of $a_{\vec{k}}$ for different $\vec{k}$'s.

Modes have zero point energy ($= \frac{1}{2}\sum n_{\vec{k}}|\vec{k}|$) where, for simplicity, the energy of mode $\vec{k}$ is taken equal to $|\vec{k}|$. This must be cut off, which we take to be at $|\vec{k}| = K + (\Delta/2)$. K would be the cut-off of the uncondensed state, a "Fermi sea" where $< n_{\vec{k}} >_{sea} = \theta(K - |\vec{k}|)$. Modes relevant to condensation are denoted $\vec{k} \in \Delta$. They lie within a spherical shell of thickness Δ about K where $K = \mathcal{O}(m_{Pl})$ and Δ is a finite fraction thereof of $\mathcal{O}(1)$. Δ is a measure of the inverse response time of the reservoir when it is perturbed by a mode through the action of Eqs (1a,1b), [9].

Up to an irrelevant subtraction the part of the kinetic energy relevant to condensation is conveniently written as

$$T_{red} = \sum_{\vec{k}\in\Delta} \tilde{\varepsilon}_{\vec{k}}\ [n_{\vec{k}} + n_{-\vec{k}} - 1] \tag{2}$$

where $\tilde{\varepsilon}_{\vec{k}} = \frac{1}{4}(|\vec{k}| - K)$.

The interactions,eqs (1), are responsible for several phenomena. Eq. (1a) gives long lasting dissipative effects, those we have invoked to maintain the 2-fluid equilibrium of vacuum. Eq. (1b) contains the sense of a static interaction energy (the term $\vec{q} = 0$ in lowest order), as well as scattering of modes by the reservoir. We shall suppose that the sum of these effects delivers a good approximation to the the true vacuum. This approximate vacuum has a filled Fermi sea of modes for $|\vec{k}| < K$. We call this unperturbed vacuum. Without mode- mode attraction induced by virtual effects such a vacuum will not be a condensate and will not possess the collective excitation we are seeking. It is the analog of a normal metal and not a superconductor.

To obtain a BCS condensate it is essential that the induced mode-mode interaction be attractive. The mechanism for this attraction is that the perturbed energy induced by the interactions, eqs (1), is due to virtual transitions from the unperturbed ground state to unperturbed excited states, hence a negative energy denominator in second order. Expressed more physically, the presence of a mode polarizes the reservoir fluid towards the space-time region where that mode is localized. (See the above discussion of Λ). A second mode is in turn attracted to this region [9].

The attraction causes Cooper pair formation, hence an instability of the unperturbed vacuum. The BCS condensate is built out of zero momentum pairs. It is constructed from the reduced hamiltonian [7].

$$H_{red} = T_{red} + \frac{1}{2} \sum_{\vec{k},\vec{k}' \in \Delta} V_{\vec{k},\vec{k}'} b_{\vec{k}} b^{\dagger}_{\vec{k}'} \tag{3}$$

where $b_{\vec{k}} = a_{\vec{k}} a_{-\vec{k}}$ and T_{red} by eq. [2]. For $V_{\vec{k},\vec{k}'} < 0$, the condensate is formed characterized by $< b_{\vec{k}} >_{cond} \neq 0$ in a small interval, μ, about K. The mass, μ, obeys an eigenvalue condition which in the approximation of a momentum independent $V_{\vec{k},\vec{k}'}$ (simplification which does no injustice to the physics here displayed)

$$1 = |V| \sum_{\vec{k} \in \Delta} \frac{1}{\sqrt{\tilde{\varepsilon}^2_{\vec{k}} + \mu^2}} \simeq N\ |V| \ln \frac{\Delta}{\mu} \tag{4}$$

(for $\mu << \Delta$). N is the density of energies, $\tilde{\varepsilon}$, in Δ. As a result μ is of the form

$$\mu = \Delta\ \exp[-1/g^2] \tag{5}$$

where the non dimensional constant g^2 is a product of the square of coupling constants multiplying 1, the square of the reservoir field, ρ, an $|$ energy denominator$|$ and N.

There are two collective excitations of the condensate [8], the massless gauge symmetry restorer (gauge symmetry being broken by $< b_k >_{cond} \neq 0$) and a scalar whose mass is μ. This is our inflaton.

To validate its candidature the inflaton must enjoy certain properties. Firstly it must be possible to build up a classical field of large amplitude. From experience of spin wave theory this should not offer difficulties. Less evident is how to

characterize the mechanism(s) for its conversion to the on-mass shell quanta of QFT (post- inflationary heating). This could involve its coupling to the mode pairs out of which it is comprised and thence to their excitation. Or it may entail the intervention of the reservoir. One must investigate precise models to elucidate these points, and to calculate μ.

The main point of this essay has been to show that a constant planckian cut-off of modes requires an elaborate mechanism for its realization. This in turn leads to a BCS condensate and an inflaton. For values of the coupling constant $g(\simeq 0.3)$, one comes upon the required $\mu \simeq (10^{-6}$–$10^{-5})m_{Pl}$. This obviates the necessity of extremely fine tuned parameters offering some grounds for encouragement.

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